Control and Optimization of Smart Grid

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Computing + Math Sciences
Electrical Engineering

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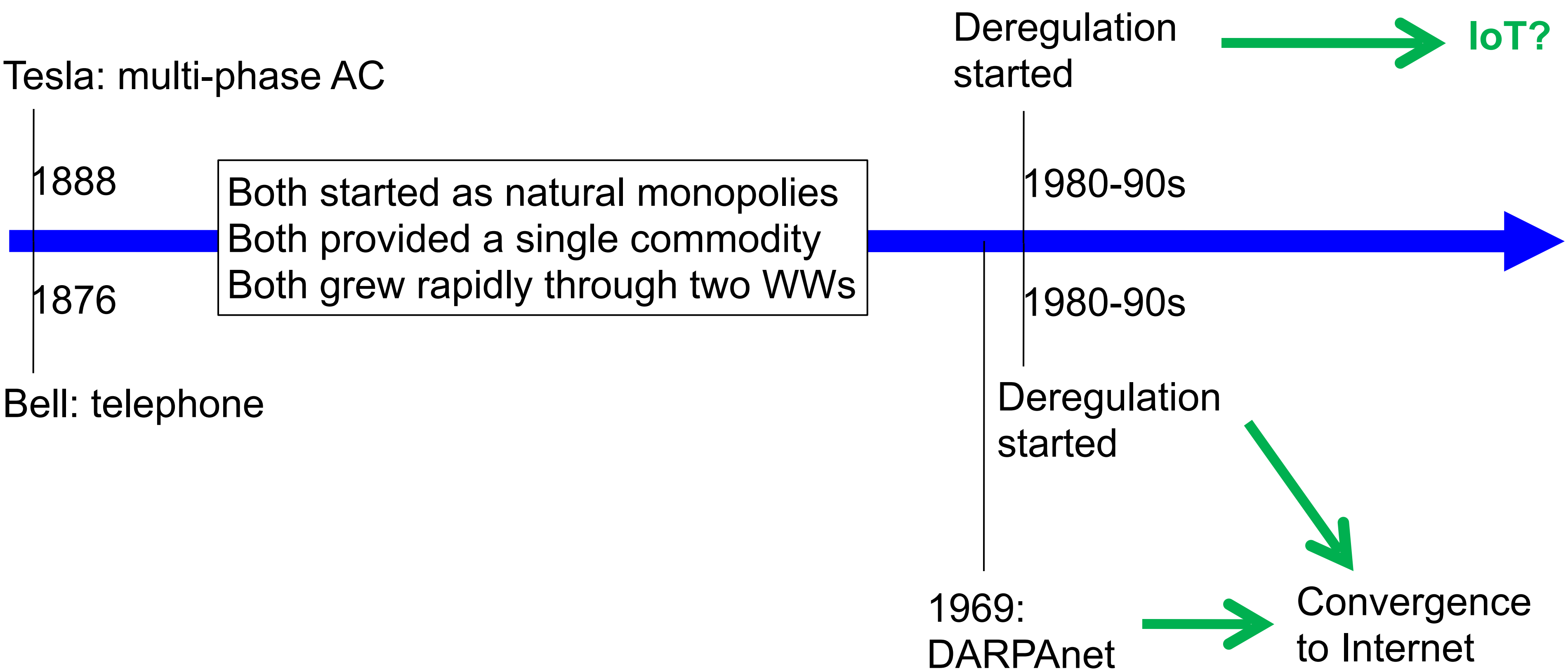
Outline

Motivation & challenges

Optimal power flow

Watershed moment

Power network will undergo similar **architectural transformation** that phone network went through in the last two decades



Watershed moment

Industries will be destroyed & created

- AT&T, MCI, McCaw Cellular, Qualcom
- Google, Facebook, Twitter, Amazon, eBay, Netflix

Infrastructure will be reshaped

- Centralized intelligence, vertically optimized
- Distributed intelligence, layered architecture

What will drive power network transformation ?

Four drivers

Proliferation of renewables

Electrification of transportation

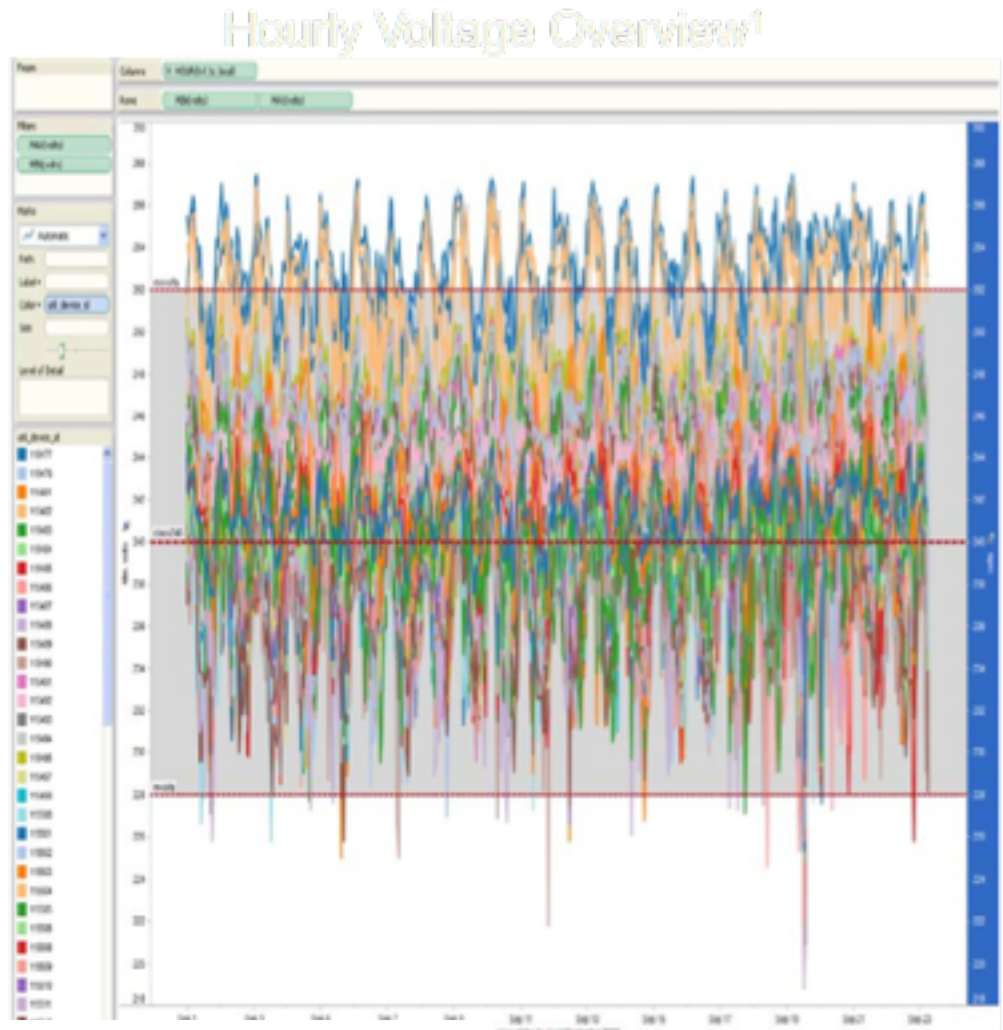
challenges

Advances in power electronics

Deployment of sensing, control, comm

enablers

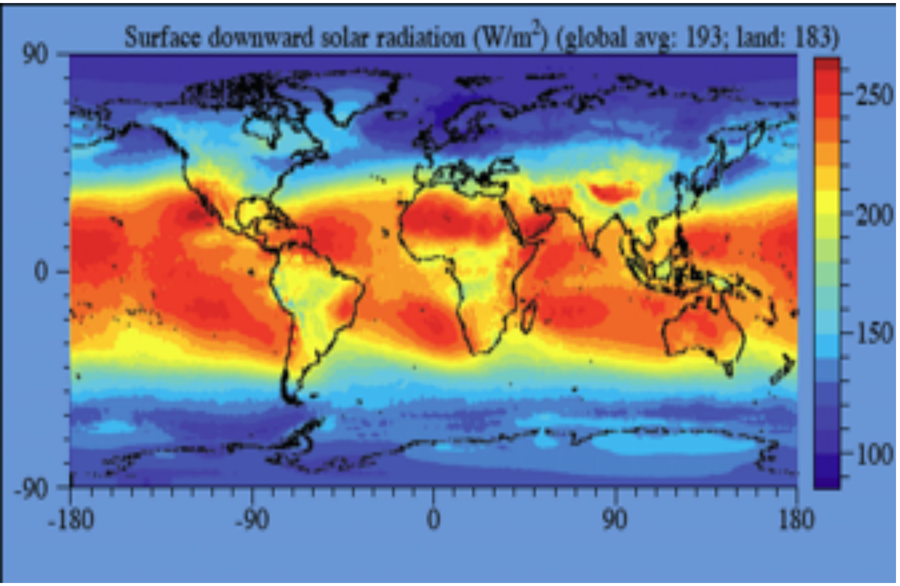
- 68 meters (residential)
- Sept 2012 (23 days)
- 240 volts
- +-5% min-228/max-252
- Hourly by meter #
- A few "high" meters
- Larger # of low meters

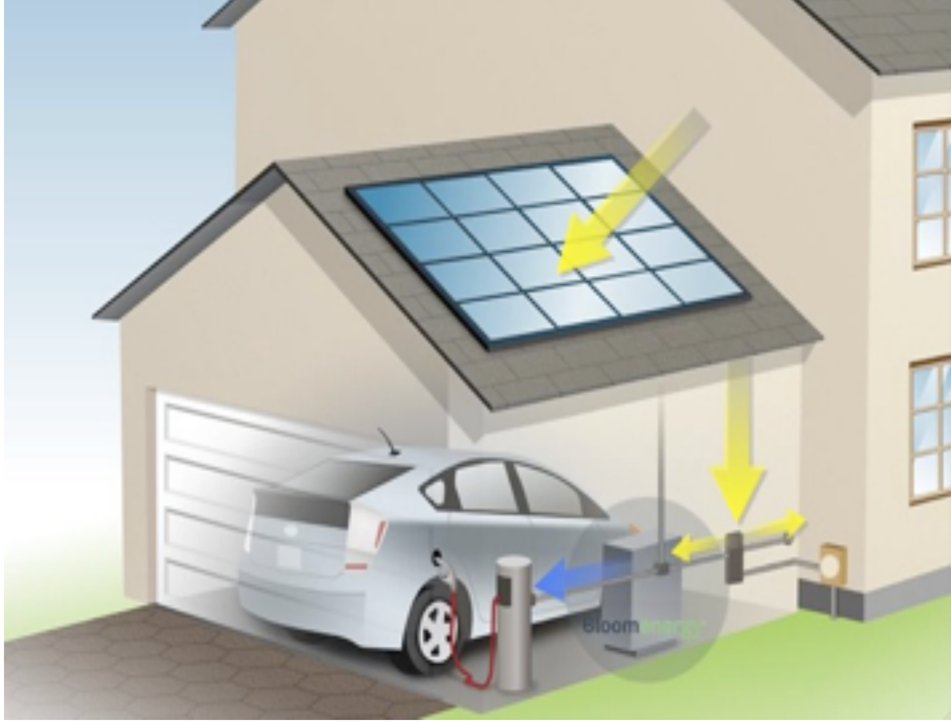


Voltage violations are quite frequent

Source: Leon Roose, University of Hawaii
Development & demo of smart grid inverters for high-penetration PV applications

Solar power over land:
> 20x world energy demand





network of
billions of active distributed
energy resources (DERs)

DER: PV, wind tb, EV, storage, smart appliances

Hawaii's solar power flare-up: Too much of a good thing?

So many private solar panels are returning power to the grid that utilities ... systems can't handle it all.

November 17, 2013

Germany's Green Energy Destabilizing Electric Grids

"Energiewende"

JANUARY 23, 2013

Power struggle: Green energy versus a grid that's not ready

Minders of a fragile national power grid say the rush to renewable energy might actually make it harder to keep the lights on.

December 02, 2013 | By Evan Halper

...volution Hiccups: Grid Instability ... Solutions

By Catalina Schröder

Sudden fluctuations in Germany's power grid are causing major dam... companies. While many of them have responded by getting their ow... help minimize the risks, they warn that companies might be forced t... with the issues fast.

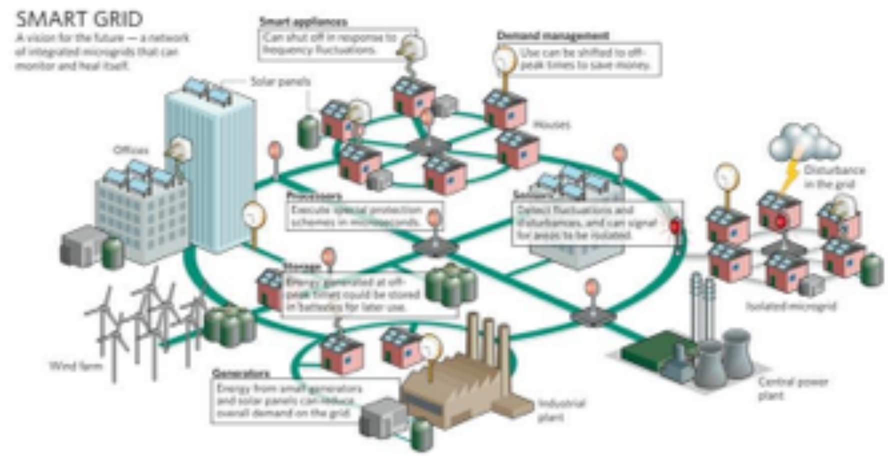
... up to us.

... Initiative

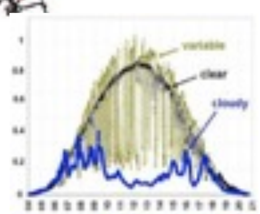
Power customers opt to go off grid

Risk: active DERs introduce rapid random fluctuations in supply, demand, power quality increasing risk of blackouts

Opportunity: active DERs enables realtime dynamic network-wide feedback control, improving robustness, security, efficiency



Caltech research: distributed control of networked DERs



- Foundational theory, practical algorithms, concrete applications
- Integrate engineering and economics
- Active collaboration with industry

Active DERs: implications

Current control paradigm works well today

- Centralized, open-loop, human-in-loop, worst-case preventive
- Low uncertainty, few active assets to control
- Schedule supplies to match loads

Future needs

- Closing the loop, e.g. real-time DR, volt/var
- Fast computation to cope with rapid, random, large fluctuations in supply, demand, voltage, freq
- Simple algorithms to scale to large networks of active DER
- Market mechanisms to incentivize

Key challenges

Nonconvexity

- Convex relaxations

Large scale

- Distributed algorithms

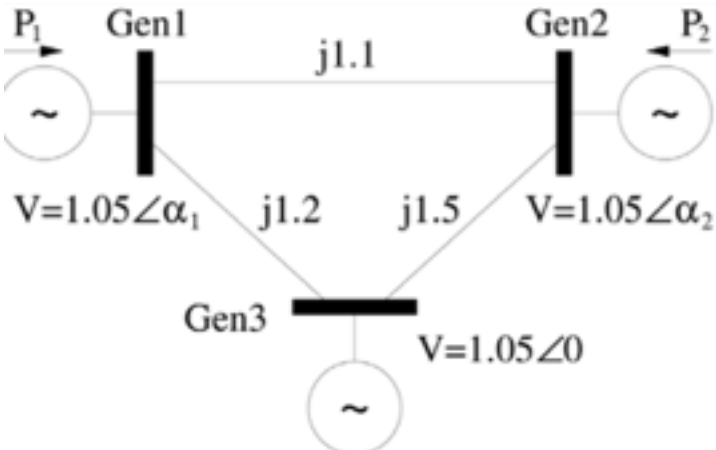
Uncertainty

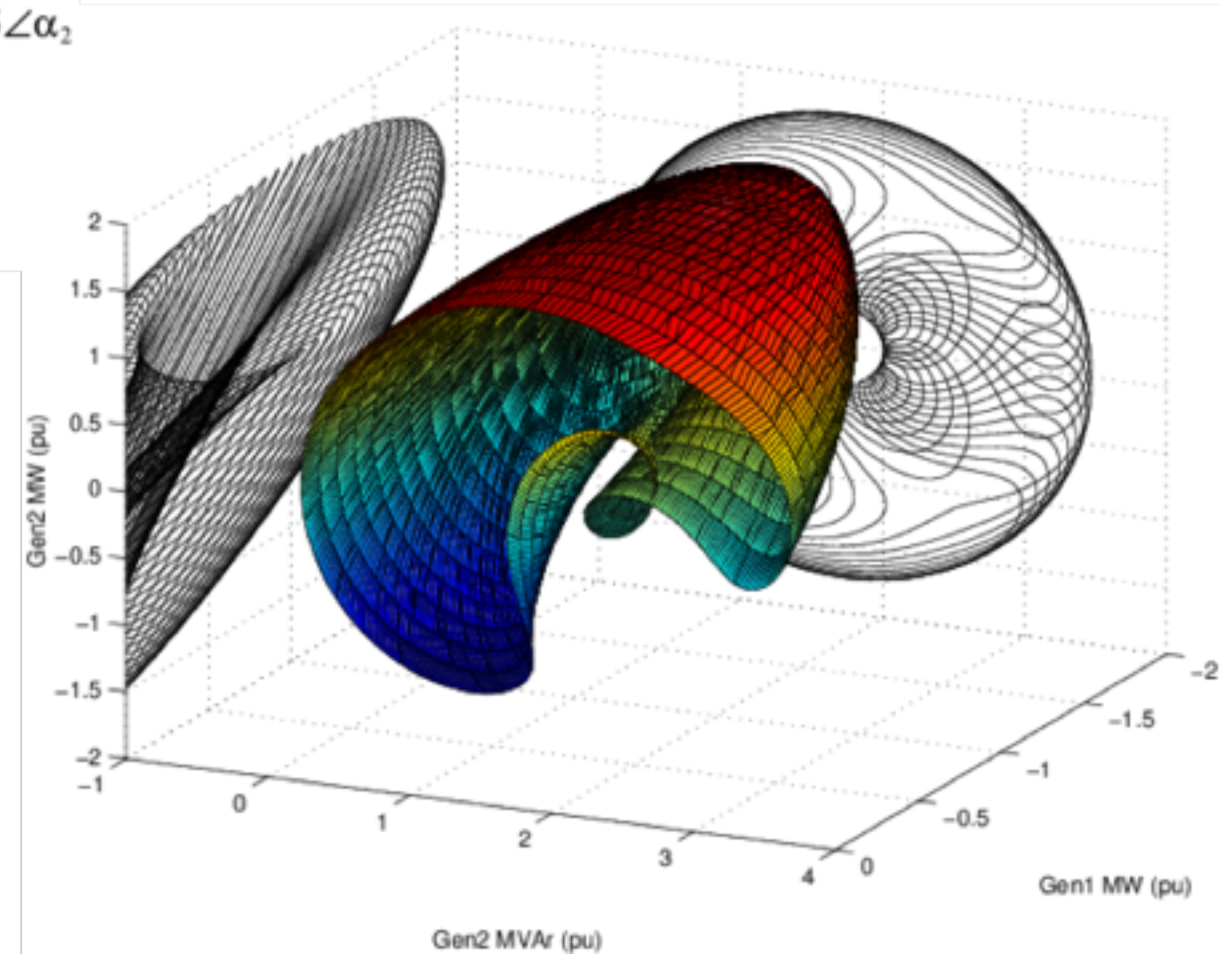
- Risk-limiting approach

Multiple timescales

- Decomposition

Nonconvexity





Ian Hiskens, Michigan

Optimal power flow (OPF)

OPF is solved routinely for

- network control & optimization decisions
- market operations & pricing
- at timescales of mins, hours, days, …

Non-convex and hard to solve

- Huge literature since 1962
- Common practice: DC power flow (LP)
- Also: Newton-Ralphson, interior point, …

Optimal power flow (OPF)

OPF underlies many applications

- Unit commitment, economic dispatch
- State estimation
- Contingency analysis
- Feeder reconfiguration, topology control
- Placement and sizing of capacitors, storage
- Volt/var control in distribution systems
- Demand response, load control
- Electric vehicle charging
- Market power analysis
- …

Nonconveity of OPF

Semidefinite relaxations of power flows

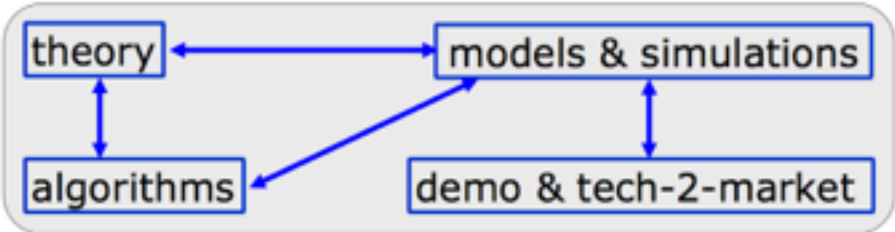
- Physical systems are nonconvex …
- … but have hidden convexity that should be exploited

Convexity is important for OPF

- Foundation of LMP, critical for efficient market theory
- Required to guarantee global optimality
- Required for real-time computation at scale

Distributed Control of Networked DER

an arpa·e GENI project



Caltech: Profs Chandy, Doyle, **Low (PI)**; Drs. Bunn, Mallada; Students: Agarwal, Cai, Chen, Farivar, Gan, Guo, Matni, Peng, Ren,Tang, You, Zhao
SCE: Auld, Castaneda, Clarke, Gooding, Montoya, Shah, **Sherick (PI)**
Newport/Caltech: DeMartini (advisor)
Alumni: Bose (Cornell), Chen (Colorado), Collins (USC), Gayme (JHU), Lavaei (Columbia), Li (Harvard), Topcu (UPenn), Xu (SUTD)

EAN

- Increase(asset(u+liza+on(and(efficiency(
- Improve(power(quality(and(stability(
- Move(data:in:mo+on(analy+cs(to(edge(

- EAN analytics and optimization
  DER placement, asset opt, analytics
- EAN enabled control
  DER co-optimization, frequency reg

Contact: Michael Enescu, co-founder CEO, enescu@alumni.caltech.edu

applications and T2M

theory

Convex relaxation of OPF: Theoretical foundation for semi-definite relaxations of power flow

OPF: $\min_{V} \ \mathrm{tr}(CVV^*)$

s. t. $\underline{s}_j \le \mathrm{tr}(Y_j^* VV^*) \le \overline{s}_j, \quad \underline{v}_j \le |V_j|^2 \le \overline{v}_j$

quadratic in V

linear in W !

SDP relaxation

$\min_{W} \ \mathrm{tr}(CW)$

s. t. $\underline{s}_j \le \mathrm{tr}(Y_j^* W) \le \overline{s}_j, \quad \underline{v}_j \le W_{jj} \le \overline{v}_j$

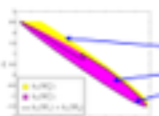
$W \succeq 0, \quad \boxed{\mathrm{rank}\ W = 1}$ ignore this (only) nonconvex constr

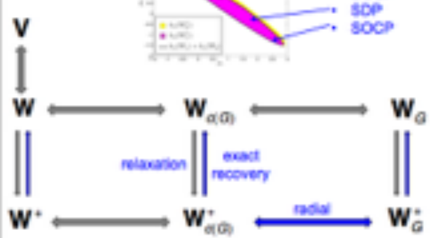
Exact relaxations: Sufficient conditions for recovering global optimum of OPF from relaxations

algorithms

Relaxation algorithms:

- single-phase balanced, multiphase unbalanced
- centralized, distributed





SDP relaxation	Chordal relaxation	SOCP relaxation
• tightest superset	• equivalent superset	• coarsest superset
• max # variables	• much faster for sparse networks	• min # variables
• slowest		• fastest

models

DER adoption model & software

- Sophisticated feedback model
- Cloud service for PV-uptake: http://etechuptake.appspot.com/

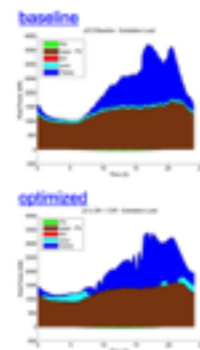
volt/var control with renewables

- SCE circuits, DER forecasts
- advanced OPF solver

simulations

Realistic simulations

- SCE feeder model, 2,000 buses
- DER: inverters, HVAC, pool pumps, EV
- Multiphase unbalanced radial



Lead: Prof Mushkin
Undergrads: Chang, Li, Yap, Zhou

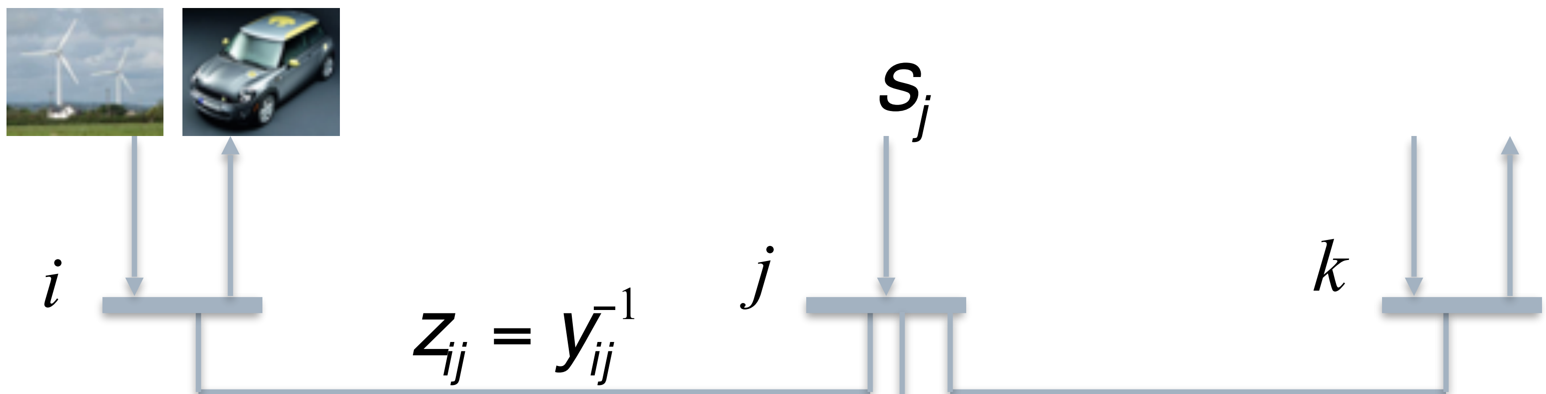
Outline

Motivation & challenges

Optimal power flow (OPF)

- problem formulation
- semidefinite relaxations
- exact relaxation

Bus injection model



admittance matrix:

$$Y_{ij} := \begin{cases} \sum_{k \sim i} y_{ik} & \text{if } i = j \\ -y_{ij} & \text{if } i \sim j \\ 0 & \text{else} \end{cases}$$

graph G: undirected

Y specifies topology of G and impedances z on lines

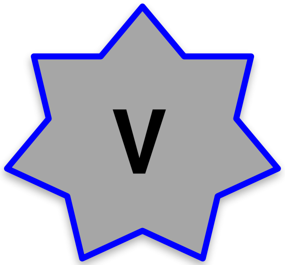
Bus injection model

In terms of V:

$$s_j = \mathrm{tr}\left(Y_j^H V V^H\right) \qquad \text{for all } j \qquad\qquad Y_j = Y^H e_j e_j^T$$

Power flow problem:

Given (Y, s) find V



isolated solutions

OPF: bus injection model

$$\min \quad \mathrm{tr}\left(CVV^H\right)$$

gen cost,
power loss

$$\text{over} \quad (V, s)$$

$$\text{subject to} \quad \underline{s}_j \le s_j \le \overline{s}_j \qquad \underline{V}_j \le |V_j| \le \overline{V}_j$$

OPF: bus injection model

$$\min \quad \operatorname{tr}\left(CVV^H\right)$$

gen cost,
power loss

$$\text{over} \quad (V, s)$$

$$\text{subject to} \quad \underline{s}_j \le s_j \le \overline{s}_j \qquad \underline{V}_j \le |V_j| \le \overline{V}_j$$

$$\boxed{s_j = \operatorname{tr}\left(Y_j^H V V^H\right)}$$

power flow equation

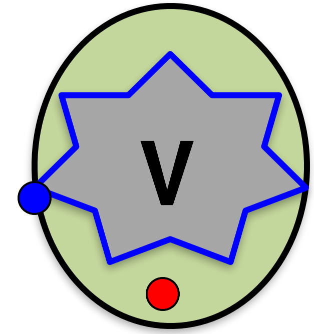
OPF: bus injection model

$$\min \quad \operatorname{tr} CVV^H$$

$$\text{subject to} \quad \underline{s}_j \le \boxed{\operatorname{tr}\left(Y_j VV^H\right)} \le \overline{s}_j \qquad \underline{v}_j \le |V_j|^2 \le \overline{v}_j$$

nonconvex QCQP
(quad constrained quad program)

Basic idea



$$\begin{aligned} \min \quad & \operatorname{tr} CVV^H \\ \text{subject to} \quad & \underbrace{\underline{s}_j \ \le\ \boxed{\operatorname{tr}\left(Y_j V V^H\right)} \ \le\ \bar{s}_j \qquad \underline{v}_j \ \le\ |V_j|^2 \ \le\ \bar{v}_j}_{\mathbf{V}} \end{aligned}$$

Approach

1. Three equivalent characterizations of $\mathbf{V}$
2. Each suggests a lift and relaxation

- What is the relation among different relaxations ?
- When will a relaxation be <u>exact</u> ?

Feasible set & SDP

$$\min \quad \operatorname{tr} CVV^H$$

$$\text{subject to} \quad \underline{s}_j \le \boxed{\operatorname{tr}\left(Y_j VV^H\right)} \le \bar{s}_j \qquad \underline{v}_j \le |V_j|^2 \le \bar{v}_j$$

quadratic in V
linear in W

Equivalent problem:

$$\min \quad \operatorname{tr} CW$$

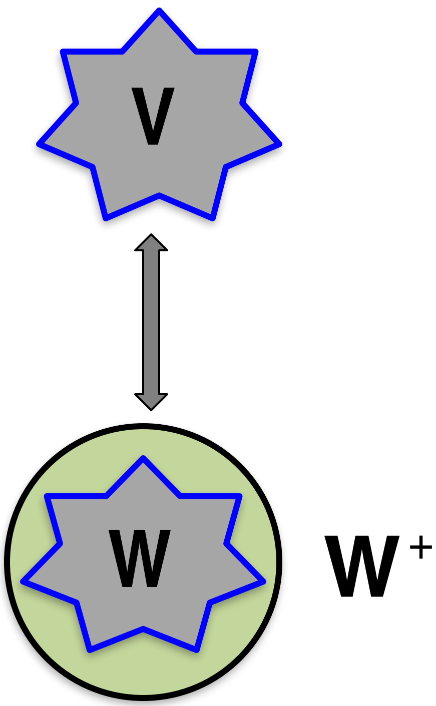
$$\text{subject to} \quad \boxed{\underline{s}_j \le \operatorname{tr}\left(Y_j W\right) \le \bar{s}_j \qquad \underline{v}_i \le W_{ii} \le \bar{v}_i}$$

$$W \ge 0, \quad \operatorname{rank} W = 1$$

convex in W
except this constraint

Equivalent feasible sets

$$\mathbf{V} := \{V: \underline{\text{quadratic}} \text{ constraints } \}$$



QCQP: n variables

SDP: n^2 vars !

$$\mathbf{W} := \{W: \underline{\text{linear}} \text{ constraints } \} \cap \{W \geq 0 \ \cancel{\text{rank-1}}\}$$

idea: $W = VV^H$

Feasible set

only $n+2m$ vars !

$$\overbrace{\sum_{k:k\sim j} y_{jk}^H\left(|V_j|^2 - V_j V_k^H\right)}^{\text{linear in } (W_{jj},\, W_{jk})} : \ \textbf{only}\ \overbrace{|V_j|^2}^{W_{jj}} \ \text{and}\ \overbrace{V_j V_k^H}^{W_{jk}}$$

corresponding to edges (j,k) in G!

$$\begin{aligned} \min \quad & \operatorname{tr} CVV^H \\ \text{subject to} \quad & \underbrace{\underline{s}_j \le \boxed{\operatorname{tr}\left(Y_j VV^H\right)} \le \bar{s}_j \qquad \underline{v}_j \le |V_j|^2 \le \bar{v}_j}_{\mathbf{V}} \end{aligned}$$

Feasible set

only $n+2m$ vars !

linear in $\left(W_{jj}, W_{jk}\right)$ $\leftarrow$ W_{jj} W_{jk}

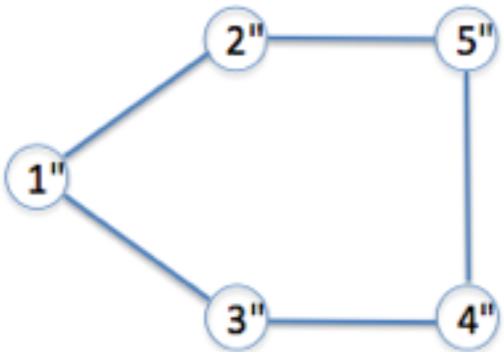
$$\underbrace{\sum_{k:k\sim j} y_{jk}^H \left(|V_j|^2 - V_j V_k^H\right)}: \textbf{ only } \underbrace{|V_j|^2} \text{ and } \underbrace{V_j V_k^H}$$

partial matrix W_G defined on G

$$W_G := \left\{[W_G]_{jj}, [W_G]_{jk} \,\middle|\, j, jk \in G\right\}$$

Kirchhoff's laws depend directly only on W_G

Example



G

$$W_G = \begin{bmatrix} W_{11} & W_{12} & W_{13} & & \\ W_{21} & W_{22} & & & W_{25} \\ W_{31} & & W_{33} & W_{34} & \\ & & W_{43} & W_{44} & W_{45} \\ & W_{52} & & W_{54} & W_{55} \end{bmatrix}$$

Want to solve for W_G

$n+2m$ variables

$$W = \begin{bmatrix} W_{11} & W_{12} & W_{13} & W_{14} & W_{15} \\ W_{21} & W_{22} & W_{23} & W_{24} & W_{25} \\ W_{31} & W_{32} & W_{33} & W_{34} & W_{35} \\ W_{41} & W_{42} & W_{43} & W_{44} & W_{45} \\ W_{51} & W_{52} & W_{53} & W_{54} & W_{55} \end{bmatrix}$$

SDP solves for $W \in \mathbf{C}^{n^2}$

n^2 variables

Feasible sets

OPF $\quad \mathbf{V} := \left\{ V \,\middle|\, \underline{s}_j \le \mathrm{tr}\left(Y_j V V^H\right) \le \bar{s}_j, \quad \underline{v}_j \le |V_j|^2 \le \bar{v}_j \right\}$

SDP

$$\mathbf{W} := \left\{ W \,\middle|\, \underline{s}_j \le \mathrm{tr}\left(Y_j W\right) \le \bar{s}_j, \ \underline{v}_j \le W_{jj} \le \bar{v}_j \right\} \cap \left\{ W \ge 0, \text{rank-1} \right\}$$

first idea:

$$\mathbf{W}_G := \left\{ W_G \,\middle|\, \underline{s}_j \le \mathrm{tr}\left(Y_j W_G\right) \le \bar{s}_j, \ \underline{v}_j \le [W_G]_{jj} \le \bar{v}_j \right\} \cap \left\{ W_G \ge 0, \text{rank-1} \right\}$$

W_G is equivalent to V when G is **chordal**

Not equivalent otherwise …

Equivalent feasible sets

$$\mathbf{W}_{c(G)} := \left\{ W_{c(G)} : \ \underline{\text{linear}} \text{ constraints } \right\} \cap \left\{ W_{c(G)} \geq 0 \text{ rank-1} \right\}$$

$$\text{idea: } \ W_{c(G)} = \left(VV^H \text{ on } c(G) \right)$$

$$\mathbf{W} := \left\{ W : \ \underline{\text{linear}} \text{ constraints } \right\} \cap \left\{ W \geq 0 \text{ rank-1} \right\}$$

$$\text{idea: } \ W = VV^H$$

Equivalent feasible sets

$$\mathbf{W}_G := \left\{ W_G : \underline{\text{linear}} \text{ constraints } \right\}$$

idea: $W_G = \left(VV^H \text{ only on } G \right)$

$$\mathbf{W}_{c(G)} := \left\{ W_{c(G)} : \underline{\text{linear}} \text{ constraints } \right\} \cap \left\{ W_{c(G)} \geq 0 \text{ rank-1} \right\}$$

idea: $W_{c(G)} = \left(VV^H \text{ on } c(G) \right)$

$$\mathbf{W} := \left\{ W : \underline{\text{linear}} \text{ constraints } \right\} \cap \left\{ W \geq 0 \text{ rank-1} \right\}$$

idea: $W = VV^H$

Equivalent feasible sets

$$\mathbf{W}_G := \{W_G : \underline{\text{linear}}\text{ constraints }\} \cap \left\{\begin{matrix} W(j,k) \geq 0 \text{ rank-1}, \\ \text{cycle cond on } \angle W_{jk} \end{matrix}\right\}$$

idea: $W_G = \left(VV^H \text{ only on } G\right)$

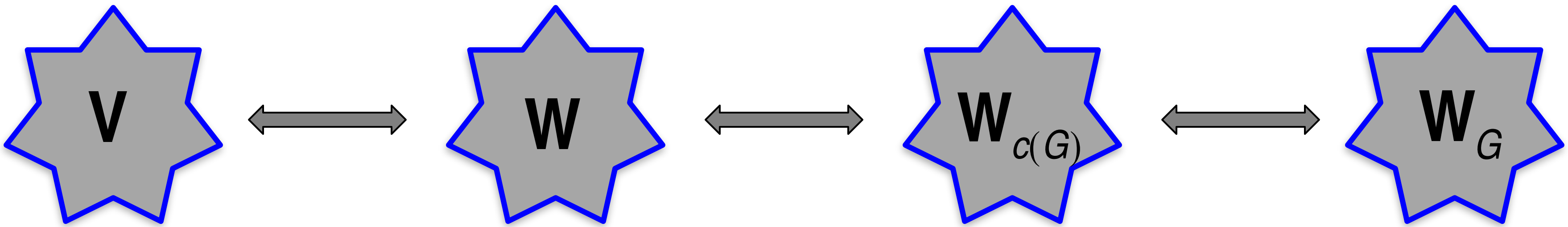
$$\mathbf{W}_{c(G)} := \{W_{c(G)} : \underline{\text{linear}}\text{ constraints }\} \cap \{W_{c(G)} \geq 0 \text{ rank-1}\}$$

idea: $W_{c(G)} = \left(VV^H \text{ on } c(G)\right)$

$$\mathbf{W} := \{W : \underline{\text{linear}}\text{ constraints }\} \cap \{W \geq 0 \text{ rank-1}\}$$

idea: $W = VV^H$

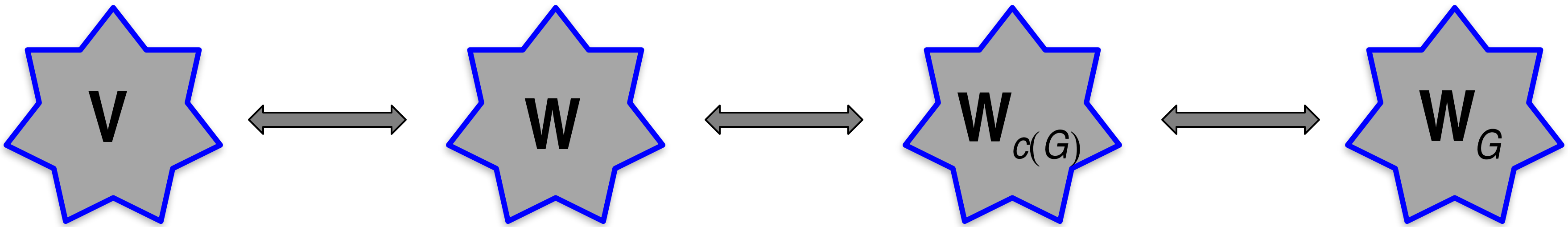
Equivalent feasible sets



<u>**Theorem:**</u> $\mathbf{V} \equiv \mathbf{W} \equiv \mathbf{W}_{c(G)} \equiv \mathbf{W}_{G}$

Bose, Low, Chandy Allerton 2012
Bose, Low, Teeraratkul, Hassibi TAC2014

Equivalent feasible sets



Theorem: $\mathbf{V} \equiv \mathbf{W} \equiv \mathbf{W}_{c(G)} \equiv \mathbf{W}_G$

Given $W_G \in \mathbf{W}_G$ or $W_{c(G)} \in \mathbf{W}_{c(G)}$ there is unique completion $W \in \mathbf{W}$ and unique $V \in \mathbf{V}$

Can minimize cost over **any** of these sets, but …

Equivalent feasible sets

$$\mathbf{W}_G := \left\{ W_G : \underline{\text{linear}} \text{ constraints} \right\} \cap \left\{ \begin{matrix} W(j,k) \geq 0 \ \cancel{\text{rank-1}}, \\ \cancel{\text{cycle cond on } \angle W_{jk}} \end{matrix} \right\}$$

idea: $W_G = \left(VV^H \text{ only on } G \right)$

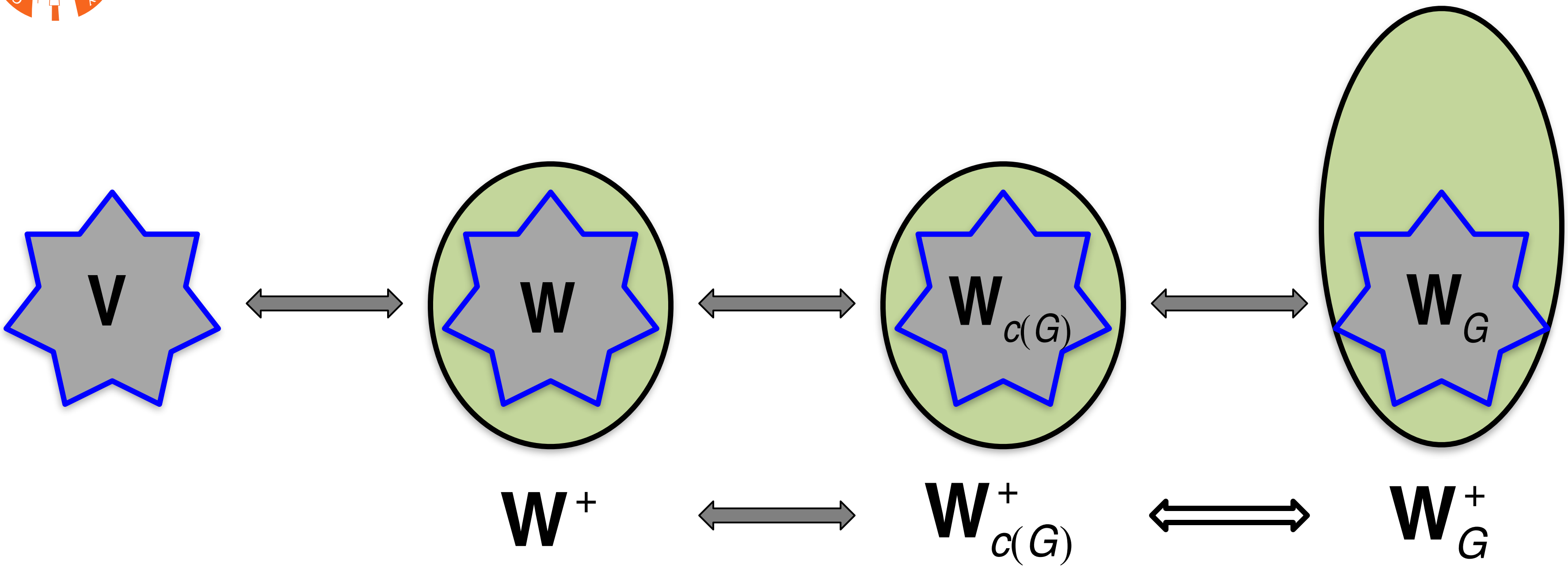
$$\mathbf{W}_{c(G)} := \left\{ W_{c(G)} : \underline{\text{linear}} \text{ constraints} \right\} \cap \left\{ W_{c(G)} \geq 0 \ \cancel{\text{rank-1}} \right\}$$

idea: $W_{c(G)} = \left(VV^H \text{ on } c(G) \right)$

$$\mathbf{W} := \left\{ W : \underline{\text{linear}} \text{ constraints} \right\} \cap \left\{ W \geq 0 \ \cancel{\text{rank-1}} \right\}$$

idea: $W = VV^H$

Relaxations

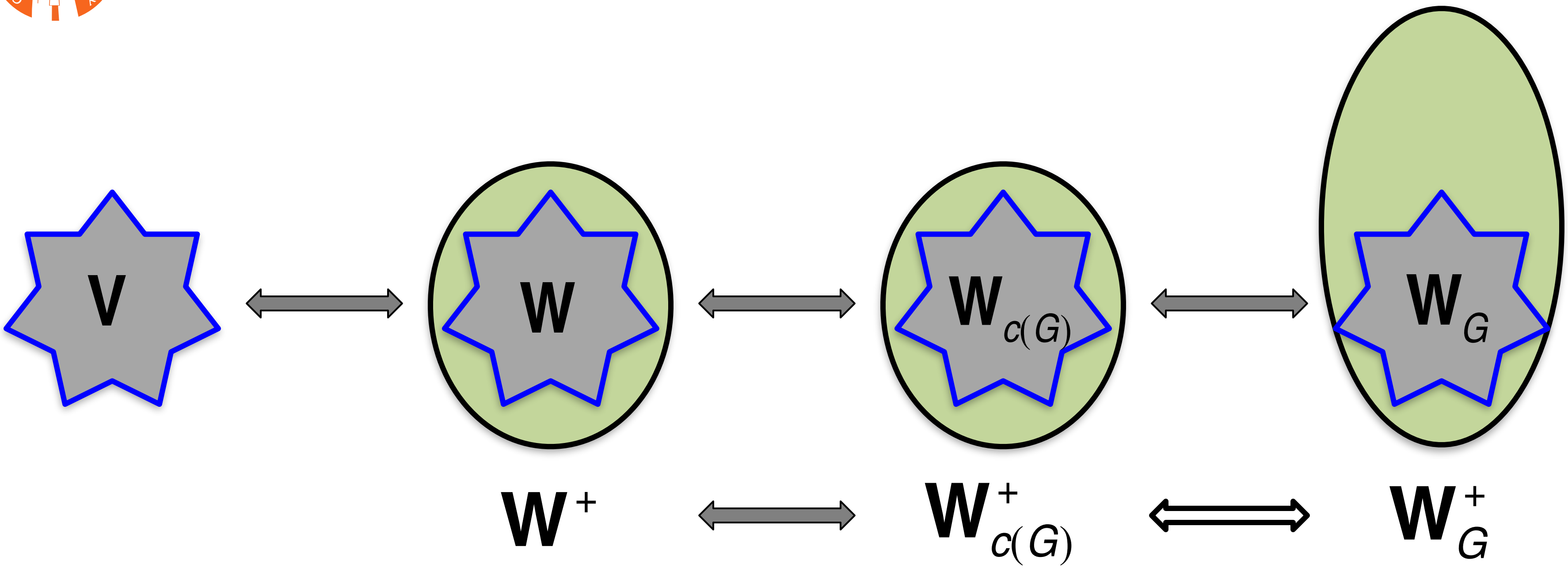


<u>Theorem</u>

- Radial G : $\mathbf{V} \subseteq \mathbf{W}^+ \cong \mathbf{W}^+_{c(G)} \cong \mathbf{W}^+_G$
- Mesh G : $\mathbf{V} \subseteq \mathbf{W}^+ \cong \mathbf{W}^+_{c(G)} \subseteq \mathbf{W}^+_G$

Bose, Low, Chandy Allerton 2012
Bose, Low, Teeraratkul, Hassibi TAC2014

Relaxations



<u>Theorem</u>

- Radial G : $\mathbf{V} \subseteq \mathbf{W}^+ \cong \mathbf{W}^+_{c(G)} \cong \mathbf{W}^+_G$
- Mesh G : $\mathbf{V} \subseteq \mathbf{W}^+ \cong \mathbf{W}^+_{c(G)} \subseteq \mathbf{W}^+_G$

For radial networks: always solve SOCP !

Recap: semidef relaxations

OPF

$$\min_{V} \; C(V) \quad \text{subject to} \quad V \in \mathbf{V}$$

OPF-sdp:

$$\min_{W} \; C(W_G) \quad \text{subject to} \quad W \in \mathbb{W}^+$$

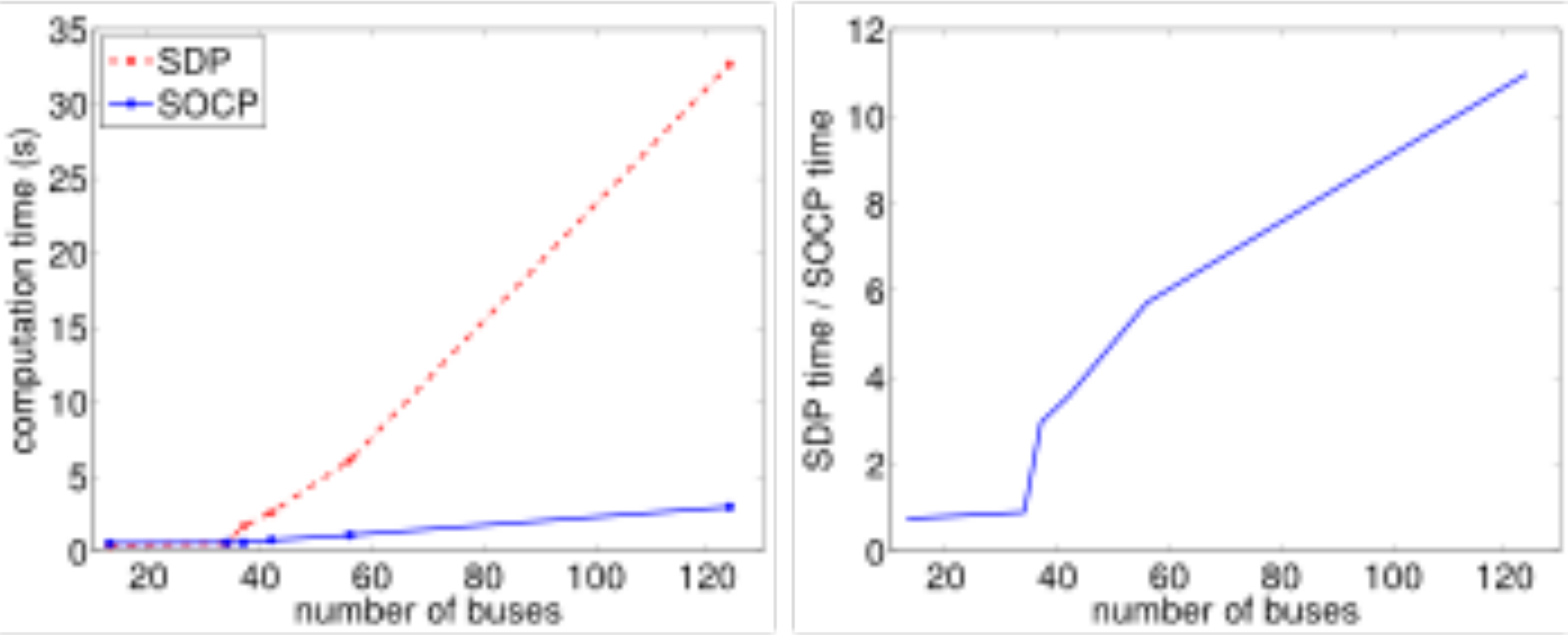
OPF-ch:

$$\min_{W_{c(G)}} \; C(W_G) \quad \text{subject to} \quad W_{c(G)} \in \mathbb{W}^+_{c(G)}$$

OPF-socp:

$$\min_{W_G} \; C(W_G) \quad \text{subject to} \quad W_G \in \mathbb{W}^+_G$$

SOCP more efficient than SDP



Relaxations are exact in all cases

- IEEE networks: IEEE 13, 34, 37, 123 buses (0% DG)
- SCE networks 47 buses (57% PV), 56 buses (130% PV)
- Single phase; SOCP using BFM
- Matlab 7.9.0.529 (64-bit) with CVX 1.21 on Mac OS X 10.7.5 with 2.66GHz Intel Core 2 Due CPU and 4GB 1067MHz DDR3 memory

OPF: extensions

semidefinite relaxations

applications

distributed OPF

Kim, Baldick 1997
Dall'Anese et al 2012
Lam et al 2012
Kraning et al 2013
Devane, Lestas 2013
Sun et al 2013
Li et al 2013
Peng, Low 2014

B&B, rank min, QC relaxation,

Phan 2012
Gopalakrishnan 2012
Louca et al 2013
Hijazi et al 2013

multiphase unbalanced

Dall'Anese et al 2012
Gan, Low 2014

exactness

ext refs in Low TCNS 2014

Louca et al 2014

moment/SoS, based relaxation

Molzahn, Hiskens 2014
Josz et al 2014
Ghaddar et al 2014